MULTI-OBJECTIVE INVENTORY MODEL OF DETERIORATING ITEMS WITH SOME CONSTRAINTS IN AN INTUITIONISTIC FUZZY ENVIRONMENT

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Abstract:

In this paper, realistic multi-item inventory models without shortages for deteriorating items with stock dependent demand have been formulated in Intuitionistic fuzzy environment. Here, total average cost, warehouse space are assumed to be imprecise in nature. The objectives of maximizing the average profit and minimizing the average deterioration cost are also intuitionistic fuzzy in nature. The impreciseness of the above constraints and objective goals have been expressed by intuitionistic fuzzy linear membership and non-membership functions.

The model have been solved by transforming the problem into two different problems, namely, Intuitionistic Fuzzy Non-linear Programming Problem (IFNLPP) and Weighted Intuitionistic Fuzzy Additive Goal Programming Problem (WIFAGPP) modified / developed for different weights. The model has been illustrated by numerical data. The optimum results for different objectives are obtained for different weights and illustrates the results.

Keywords: Intuitionistic fuzzy Non-linear Programming, Weighted Intuitionistic Fuzzy Additive Goal Programming, optimization, Inventory, Multi-objective

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1 Introduction

Since the development of EOQ model by Harris[1], a lot of research work on inventory control system has been reported in the literature(c.f. Arrow, et. al.[2], Naddor [3], Worell and Hall[4]). In the classical inventory models, normally static lot size models are formulated. But, because of the manufacturing environment, the static models are inadequate in analyzing the behavior of such systems and in deriving the optimal policies for their control. Moreover, it is usually observed in the market that sales of the fashionable goods, electronic gadgets, seasonable products, food-grains, etc., increase with time. For these reasons, dynamic models of production-inventory / inventory systems have been considered and solved by some researchers (cf. Padmanabhan and Vrat [5,6], Bendaya and Rauof [7], Hariga and Benkherouf [8], etc). In these models, demand and/or production are assumed to be continuous functions of time.

Deterministic optimization problems are well studied, but they are very limited and in many cases they do not represent exactly the real problem. Usually, it is difficult to describe the constraints of an optimization problem in crisp equality and / or inequality relations. In reality, a small violation of a given constraint is admissible and it can lead to a more efficient solution of the real problem. Objective formulation represents a subjective estimation of a possible effect of a given value of the objective function. In the last three decades optimization problems have been investigated in the sense of fuzzy set theory (c.f. Tanaka and Asai[9], Mahapatra and Maiti (10,11], etc.). The fuzzy optimization formulations are more flexible and allow to find solutions which are more adequate to the real problem. The principle of fuzzy optimization problems are critically studied by Angelov[12,13].

In general, decision making under uncertainty employs the expected / average profit maximization objective and / or expected / average cost minimization objective. These objectives do not always reflect what managers / decision makers attempt to do in practice. Indeed, managers / decision makers of different business / firm houses set targets at the beginning of each year and they are often more interested in optimizing the achievement of the target returns than in optimizing the objectives.

On the other hand, fuzzy set theory has been widely developed and various modifications and generalizations have appeared. One of them is the concept of intuitionistic fuzzy (IF) sets (Atanassov[14]). They consider not only the degree of membership to a given set, but also the degree of rejection such that the sum of both values is less than one. Applying this concept, it is possible to reformulate the optimization problem by using degrees of rejection of constraints and values of the objective which are non-admissible. The degree of acceptance and of rejection can be arbitrary, but the sum of both have to be less than or equal to one. An approach to solve such intuitionistic fuzzy multi-objective non-linear optimization problem (IFMONLOP) is proposed and a simple illustrative example is considered in this paper.

In this paper, realistic multi-item inventory models without shortages for deteriorating items with stock dependent demand have been formulated in Intuitionistic fuzzy environment. Here, total average cost, warehouse space are assumed to be imprecise in nature. The objectives of maximizing the average profit and minimizing the average deterioration cost are also intuitionistic fuzzy in nature. The impreciseness of the above constraints and objective goals have been expressed by intuitionistic fuzzy linear membership and non-membership functions.

The model have been solved by transforming the problem into two different problems, namely, Intuitionistic Fuzzy Non-linear Programming Problem (IFNLPP) and Weighted Intuitionistic Fuzzy Additive Goal Programming Problem (WIFAGPP) modified / developed for different weights. The model have been illustrated by numerical data. An efficient algorithm has also been prescribed for solving above type of problems. The optimum results for different objectives are obtained for different weights and illustrates the results.

2 Proposed Inventory Model

A multi-objective inventory model of deteriorating items with infinite rate of replenishment, without shortage, stock-dependent demand and limited storage capacity is developed under the following notations and assumptions.

Assumptions:

• The demand rates are constant and known

• Holding cost applies on good units only

• Shortages are not allowed

• Rate of deterioration is constant and there is no repair or replacement for the deteriorated units

• The replenishment rate is instantaneous without lead time

Notations:

For $i-$th item,

$q_i(t)$ = inventory level at any time t

Q_i = Order quantity for $i-$th item

θ_i = Constant rate of deterioration, $0 \le \theta_i \le 1$

p_i = Purchasing price of each unit

s_i = Selling price of each unit

h_i = holding cost per unit quantity per unit time

u_i = Set-up cost

T_i = Time period for each cycle

$D_i(q_i)$ = quantity of demand at time $t = \alpha_i + \beta_i q_i(t)$, $\alpha_i (\neq 0)$ and $\beta_i (0 < \beta_i < 1)$ being constants

A_i = Required storage space per unit quantity

n = Number of items

$AP(Q_i)$ = Total average profit of all items

$ADC(Q_i)$ = Total deterioration cost of all items

$TC(Q_i)$ = Total average cost of all items

W = Available storage space for all items

C = Upper limit on total average cost for all items

2.1 Mathematical formulation

Let us consider an inventory system starts with maximum inventory Q_i of $i-$th item at the beginning of time period T_i and inventory level declines due to demand of the customers and deterioration of the item $(i = 1, 2, ..., n)$. If $q_i(t)$ is the inventory level of $i-$th item at any time t of a time period, then the differential equation governing stock status of $i-$th item is given by the following equation:

$$\frac{dq_i}{dt} = -D_i(q_i) - a_i q_i \ for\ 0 \le q_i \le Q_i, (i = 1, 2, ..., n) \quad (1)$$

So, the length of the cycle of the $i-$th item is:

$$T_i = \int_0^{Q_i} \frac{dq_i}{D_i(q_i) + a_i q_i} = \int_0^{Q_i} \frac{dq_i}{\alpha_i + \beta_i q_i + a_i q_i} = \frac{1}{a_i + \beta_i} \ln(\frac{\alpha_i + (\beta_i + a_i)Q_i}{\alpha_i}) \quad (2)$$

The number of holding units in each cycle for the $i-$th item is:

$$G_i(Q_i) = \int_0^{Q_i} \frac{q_i dq_i}{\alpha_i + \beta_i q_i + a_i q_i} = \frac{Q_i}{a_i + \beta_i} = \frac{Q_i}{(a_i + \beta_i)^2} \ln(\frac{\alpha_i + (\beta_i + a_i)Q_i}{\alpha_i}) \quad (3)$$

The number of deteriorating units of the $i-$th item is: $\theta_i G_i(Q_i)$ (4)

The net revenue, for the $i-$th item is: $N_i(Q_i) = (s_i - p_i)Q_i - s_i\theta_i G_i(Q_i)$ (5)

The total average cost of the $i-$th item is: $TC_i(Q_i) = \frac{1}{T_i}(p_i Q_i + C_{1i} G_i(Q_i) - C_{3i})$ (6)

Hence, the problem is to maximize total profit, minimize total deterioration cost subject to limitation of available storage area and total average cost as below :

$$Max\ APF(Q_i) = \sum_{i=1}^{n} \frac{1}{T_i}(N_i(Q_i) - C_{1i} G_i(Q_i) - C_{3i}),$$

$$Min\ ADC(Q_i) = \sum_{i=1}^{n} \frac{1}{T_i} \theta_i G_i(Q_i) p_i$$

$$subject\ to \sum_{i=1}^{n} A_i Q_i \le W, \sum_{i=1}^{n} TC_i(Q_i) \le C$$

$$Q_i > 0, i = 1, 2, ..., n. \quad (7)$$

Intuitionistic Fuzzy Inventory model:

When the above profit goal, deterioration cost, storage area and inventory costs become intuitionistic fuzzy, the above problem becomes a multi-objective intuitionistic fuzzy inventory problem as below:

$$\tilde{Max}\ \ APF(Q_i) = \sum_{i=1}^{n} \frac{1}{T_i}(N_i(Q_i) - C_{1i}G_i(Q_i) - C_{3i}),$$

$$\tilde{Min}\ ADC(Q_i) = \sum_{i=1}^{n} \frac{1}{T_i}\theta_i G_i(Q_i)\, p_i$$

$$subject\ to\ \sum_{i=1}^{n} A_i Q_i \le \tilde{W}, \sum_{i=1}^{n} TC_i(Q_i) \le \tilde{C}$$

$$Q_i > 0, i = 1, 2, ..., n. \tag{8}$$

Here (̃)(*tilde*) denotes that the corresponding item is an intuitionistic fuzzy objective/ parameter.

3 Mathematical Preliminaries

Definition: Fuzzy Set

A fuzzy set A in a universal set X is defined as $A = \{< x, \mu_A(x) > / x \in X\}$, where $\mu_A : X \to [0,1]$ is a mapping called the membership function of the fuzzy set A.

Definition: Intuitionistic Fuzzy Set (IFS)

An IFS A in X is an expression given by $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ with the condition $\mu_A(x) + \mu_A(x) \le 1$ for all $x \in X$. Here $\mu_A(x)$ and $\nu_A(x)$ are called the degree of acceptance (membership) and rejection (non-membership) of the element x to the set A.

An element x of X is called significant with respect to a fuzzy subset A of X if the degree of membership $\mu_A(x) > 0.5$, otherwise it is insignificant. We see that for a fuzzy subset A both the degree of membership $\mu_A(x)$ and non-membership $\nu_A(x) = 1 - \mu_A(x)$ can not be significant. Further for an IFS $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$, it is observed that $\mu_A(x) + \nu_A(x) \le 1$, for all $x \in X$ and hence $\mu_A(x) \wedge \nu_A(x) \le 0.5$ for all $x \in X$.

Now, we define a generalised intuitionistic fuzzy set (GIFS).

Definition: Generalised intuitionistic Fuzzy Set (GIFS)

Let E be a fixed set. A generalised intuitionistic fuzzy set A of E is an object having the form $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$, where the function $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define respectively the degree of membership and degree of non-membership of the element $x \in E$ to the set A, which is a subset of E and for every $x \in E$ satisfy the condition $\mu_A(x) \wedge \nu_A(x) \le 0.5,$ for all $x \in E$.

This condition is called generelised intuitionistic condition (GIC). In fact all GIFS is IFS but all IFS is not GIFS.

4 Mathematical formulation of Intuitionistic Fuzzy Multi-objective non-linear Optimization Problem (IFMONLOP) :

In general, an optimization problem includes objective(s) and constraints. In fuzzy optimization problems, the objective(s) and/or constraints or parameters and relations are represented by fuzzy sets. These fuzzy sets explain the degree of satisfaction of the respective condition and are expressed by their membership functions.

Crisp Multi-objective Non-linear Programming Problem: Let us consider a multi-objective non-linear optimization problem (MONLOP) as follows.

$$minimize\ f_i(x), i = 1, 2, ..., p \tag{9}$$

$$subject\ to\ g_j(x) \le 0,\ j = 1, 2, ..., q. \tag{10}$$

where x denotes unknowns, $f_i(x)$ denotes objective functions, $g_j(x)$ denotes constraints (inequalities), p denotes the number of objectives, q denotes the number of constraints. Here, some or all of the objectives and / or constraints are non-linear.

The solutions of crisp multi-objective non-linear optimization problem satisfy all constraints exactly and they are termed as Pareto optimal solutions.

Some basic definitions on solutions of multi-objective non-linear programming problem are given below.

Definition: The multi-objective optimization problem is convex if all the objective functions and the feasible region are convex.

Definition (Complete Optimal Solution): x^* is said to be a complete optimal solution to the MONLOP if and only if there exist x^* satisfying (10) such that $f_i(x^*) \le f_i(x)$, for $i = 1, 2, ..., p$ and for all $\forall x$ satisfying (10).

In general, the objective functions of the MONLP conflict with each other, a complete optimal solution does not always exist and so Pareto (or non dominated) optimality concept is introduced.

Definition(Pareto Optimal Solution): x^* is said to be a Pareto optimal solution to the MONLOP iff there does not exist another x satisfying (10) such that $f_i(x) \le f_i(x^*)$ for all $i(i = 1, 2, ..., p)$ and $f_j(x) < f_j(x^*)$ for at least one index $j(j = 1, 2, ..., p)$.

An objective vector $F^* = (f_1^*, f_2^*, ..., f_p^*)$ is Pareto-optimal if there does not exist another objective vector $F(x) = (f_1(x), f_2(x), ..., f_p(x))$ such that $f_i \le f_i^*$ for all $i = 1, 2, \cdots, p$ and $f_j < f_j^*$ for at least one index j. Therefore, F^* is Pareto-optimal if the decision vector corresponding to it is Pareto optimal.

Unless an optimization problem is convex, only locally optimal solution is guaranteed using standard mathematical programming techniques. Therefore, the concept of Pareto-optimality needs to be modified to introduce the notion of a locally Pareto-optimal solution for a non-convex problem.

Definition (Locally Pareto Optimal Solution):

x^* satisfying (10) is said to be a locally Pareto optimal solution to the MONLOP if and only if there exists an $r > 0$ such that x^* is Pareto optimal in $N(x^*, r)$, i.e, there does not exist another $x^* \in N(x^*, r)$ satisfying (10) such that $f_i(x) \le f_i(x^*)$.

Fuzzy Multi-objective Non-linear Programming Problem: In the analogous fuzzy optimization problem the degree of satisfaction of objection of objective(s) and of constraint(s) is maximized:

$$\widetilde{minimize}\, f_i(x), i = 1, 2, ..., p$$

$$subject\ to\ g_j(x) \tilde{\le} 0, j = 1, 2, ..., q. \tag{11}$$

where $\widetilde{minimize}$ denotes fuzzy minimization and $\tilde{\le}$ denotes fuzzy inequality. It is transformed via Bellman-Zadeh [1970] approach to the following crisp optimization problem.

Maximize the degree of membership (acceptance) of the objective(s) AND constraints to the respective fuzzy sets:

$$\begin{aligned} & \textit{maximize}\,\mu_i(x), x \in R^n, i = 1, 2, ..., p+q \\ & \textit{subject to } 0 \le \mu_i(x) \le 1. \end{aligned} \qquad (12)$$

where $\mu_i(x)$ denotes degree of acceptance of x to the respective fuzzy sets.

Intuitionistic Fuzzy Multi-objective Non-linear Programming Problem: In the case when the degree of rejection (non-membership) is defined simultaneously with the degree of acceptance (membership) and when both these degrees are not complementary to each other then Intuitionistic Fuzzy sets can be used as a more general and full tool for describing this uncertainty (Atanassov[14]). It is possible to represent deeply existing nuances in problem formulation defining objective(s) and constraints (or part of them) by Intuitionistic Fuzzy sets, i.e. by pair of membership $(\mu_i(x))$ and rejection $(\nu_i(x))$ functions (Atanassov[15]). An IFMONLOP problem is formulated as follows:

To maximize the degree of acceptance of Intuitionistic Fuzzy objective(s) AND constraints AND to minimize the degree of rejection of Intuitionistic Fuzzy objective(s) AND constraints:

$$\begin{aligned} & \textit{maximize}\,\mu_i(x), x \in R^n, i = 1, 2, ..., p+q, \qquad (13) \\ & \textit{minimize}\,\nu_i(x), i = 1, 2, ..., p+q, \\ & \textit{subject to } \nu_i(x) \ge 0, i = 1, 2, ..., p+q, \\ & .\mu_i(x) \ge \nu_i(x), i = 1, 2, ..., p+q, \\ & \mu_i(x) + \nu_i(x) \le 1, i = 1, 2, ..., p+q, \end{aligned}$$

where $\mu_i(x)$ denotes the degree of membership (acceptance) of x to the i -th Intuitionistic Fuzzy set and $\mu_i(x)$ denotes the degree of non-membership (rejection) of x from the i -th Intuitionistic Fuzzy set.

In case of Intuitionistic Fuzzy Multi-objective optimization problem, the membership functions and non-membership functions are used rather than the original intuitionistic fuzzy objective functions. So, the notion of Pareto optimal solutions determined in terms of objective functions may not be applicable. Hence, the concept of Intuitionistic Fuzzy Pareto (IF Pareto) optimal solution and Weak Intuitionistic Fuzzy Pareto (Weak IF Pareto) optimal solution are introduced below:

Definition (Intuitionistic Fuzzy Pareto (IF Pareto) Optimal solution): $x^* \in R^n$ is said to be an Intuitionistic Fuzzy Pareto (IF Pareto) Optimal solution to the above problem (13) if and only if there does not exist another $x \in R^n$ such that $\mu_i(f_i(x)) \geq \mu_i(f_i(x^*)), \nu_i(f_i(x)) \leq \nu_i(f_i(x^*)), \forall i$ and $\mu_j(f_j(x)) \neq \mu_j(f_j(x^*)), \nu_j(f_j(x)) \neq \nu_j(f_j(x^*)),$ for at least one $j, j \in \{1, 2, \cdots p+q\}$.

Definition (Weak Intuitionistic Fuzzy Pareto (Weak IF Pareto) Optimal solution):

$x^* \in R^n$ is said to be an Intuitionistic Fuzzy Pareto (IF Pareto) Optimal solution to the above problem (13) if and only if there does not exist another $x \in R^n$ such that $\mu_i(f_i(x)) > \mu_i(f_i(x^*)), \nu_i(f_i(x)) < \nu_i(f_i(x^*)), \forall i, i \in \{1, 2, \cdots p+q\}$.

5 Solution Techniques of IFMONLOP

IFMONLOP problems such as fuzzy optimization problems can be represented as a two-stage process which includes aggregation of constraints and objective(s) and defuzzification (maximization of aggregated function)(Angelov[12][13]). Usually the applied Bellman-Zadeh [15]'s approach for fuzzy optimization problem solving realizes min-aggregator. Conjunction of Intuitionistic Fuzzy sets is defined as (Atanassov[14]):

$$G \bigcup C = \{\langle x, \mu_G(x) \cap \mu_C(x), \nu_G(x) \cup \nu_C(x) \rangle, x \in R^n\}$$

where G denotes an Intuitionistic Fuzzy objective (gain) and C denotes an Intuitionistic Fuzzy constraint.

This operator can be easily generalized and applied to the IFMONLOP problem:

$$D = \{\langle x, \mu_D(x), \nu_D(x) \rangle, x \in R^n\}, \quad \mu_D(x) = \bigcap_{i=1}^{p+q} \mu_i(x), \nu_D(x) = \bigcap_{i=1}^{p+q} \nu_i(x)$$

where D denotes the Intuitionistic Fuzzy set of the decision.

Min-aggregator is used for conjunction and max-operator for disjunction:

$$\mu_D(x) = min_i \mu_i(x), i = 1, 2, ..., p+q, x \in R^n, \mu_D \leq \mu_i$$

$$\nu_D(x) = max_i \nu_i(x), i = 1,2,..., p+q, x \in R^n, \nu_D \geq \nu_i$$

It can be transformed to the following system of inequations:

$$\alpha \leq \mu_i, i = 1,2,..., p+q, x \in R^n,$$

$$\beta \geq \nu_i(x), i = 1,2,..., p+q, x \in R^n,$$

$$\alpha \geq \beta, \beta \geq 0, \alpha \geq 0, \alpha + \beta \leq 1.$$

where α denotes the minimal acceptable degree of objective(s) and constraints and β denotes the maximal degree of rejection of objective(s) and constraints.

Now, to solve above problem, it is transformed to any one of the following form, namely, (a) Intuitionistic Fuzzy Non-linear Programming Problem (IFNLPP) or, (b) Weighted Intuitionistic Fuzzy Additive Goal Programming Problem (WIFAGPP).

5.1 Intuitionistic Fuzzy Non-linear Programming Problem(IFNLPP)

Now the IFMONLOP problem can be transformed to the following crisp optimization problem (Angelov[17]):

$$max\ (\alpha - \beta) \tag{14}$$

$$subject\ to\ \alpha \leq \mu_i(x), i = 1,2,..., p+q, x \in R^n,$$

$$\beta \geq \nu_i(x), i = 1,2,..., p+q, x \in R^n,$$

$$\alpha \geq \beta, \beta \geq 0, \alpha \geq 0, \alpha + \beta \leq 1.$$

5.2 Weighted Intuitionistic Fuzzy Additive Goal Programming Problem (WIFAGPP)

In this case, the above problem reduced to the following crisp optimization problem.

$$Max \sum_i w_i(\mu_i(x) - \nu_i(x)), \tag{15}$$

$$subject\ to \sum_i w_i = 1, w_i \geq 0, i = 1,2,..., p+q, x \in R^n$$

$$\mu_i \geq \nu_i \geq 0, i = 1,2,..., p+q, x \in R^n$$

Here, values of different weights $(w_i) \in [0,1]$ are prescribed by the decision maker according to his / her target.

6 Intuitionistic Fuzzy Optimization Technique on Inventory Model

In intuitionistic fuzzy set theory, the fuzzy objectives and constraints are defined by their membership (i.e, degree of acceptance) and non-membership (i.e, degree of rejection) functions which may be linear and / or non-linear. Here, we assume $\mu_{APF}(Q_i), \mu_{ADC}(Q_i), \mu_W(Q_i), \mu_C(Q_i)$ be the linear membership functions and $\nu_{APF}(Q_i), \nu_{ADC}(Q_i), \nu_W(Q_i), \nu_C(Q_i)$ be the linear non-membership functions for the two objectives and two constraints respectively as below.

$$\mu_{APF}(Q_i) = \begin{cases} 0 & for\ APF(Q_i) < APF_L^A \\ 1 - \dfrac{APF_U^A - APF(Q_i)}{APF_U^A - APF_L^A} & for\ APF_L^A \le APF(Q_i) \le APF_U^A \\ 1 & for\ APF(Q_i) > APF_U^A \end{cases} \tag{16}$$

$$\mu_{ADC}(Q_i) = \begin{cases} 0 & for\ ADC(Q_i) > ADC_U^A \\ 1 - \dfrac{ADC(Q_i) - ADC_L^A}{ADC_U^A - ADC_L^A} & for\ ADC_L^A \le ADC(Q_i) \le ADC_U^A \\ 1 & for\ ADC(Q_i) < ADC_L^A \end{cases} \tag{17}$$

$$\mu_W(Q_i) = \begin{cases} 0 & for \sum_{i=1}^{n} A_i Q_i > W_U^A \\ 1 - \dfrac{\sum_{i=1}^{n} A_i Q_i - W_L^A}{W_U^A - W_L^A} & for\ W_L^A \le \sum_{i=1}^{n} A_i Q_i \le W_U^A \\ 1 & for \sum_{i=1}^{n} A_i Q_i < W_L^A \end{cases} \tag{18}$$

$$\mu_C(Q_i)=\begin{cases}0 & for \sum_{i=1}^{n} TC_i(Q_i) > C_U^A \\ 1-\dfrac{\sum_{i=1}^{n} TC_i(Q_i) - C_L^A}{C_U^A - C_L^A} & for\ C_L^A \le \sum_{i=1}^{n} TC_i(Q_i) \le C_U^A \\ 1 & for \sum_{i=1}^{n} TC_i(Q_i) < C_L^A \end{cases} \tag{19}$$

$$\nu_{APF}(Q_i)=\begin{cases}0 & for\ APF(Q_i) < APF_L^R \\ 1-\dfrac{APF_U^R - APF(Q_i)}{APF_U^R - APF_L^R} & for\ APF_L^R \le APF(Q_i) \le APF_U^R \\ 1 & for\ APF(Q_i) > APF_U^R \end{cases} \tag{20}$$

$$\nu_{ADC}(Q_i)=\begin{cases}0 & for\ ADC(Q_i) > ADC_U^R \\ 1-\dfrac{ADC(Q_i) - ADC_L^R}{ADC_U^R - ADC_L^R} & for\ ADC_L^R \le ADC(Q_i) \le ADC_U^R \\ 1 & for\ ADC(Q_i) < ADC_L^R \end{cases} \tag{21}$$

$$\nu_W(Q_i)=\begin{cases}0 & for \sum_{i=1}^{n} A_i Q_i > W_U^R \\ 1-\dfrac{\sum_{i=1}^{n} A_i Q_i - W_L^R}{W_U^R - W_L^R} & for\ W_L^R \le \sum_{i=1}^{n} A_i Q_i \le W_U^R \\ 1 & for \sum_{i=1}^{n} A_i Q_i < W_L^R \end{cases} \tag{22}$$

$$\nu_C(Q_i)=\begin{cases}0 & for \sum_{i=1}^{n}TC_i(Q_i)>C_U^R\\ 1-\dfrac{\sum_{i=1}^{n}TC_i(Q_i)-C_L^R}{C_U^R-C_L^R} & for\ C_L^R\le\sum_{i=1}^{n}TC_i(Q_i)\le C_U^R\\ 1 & for \sum_{i=1}^{n}TC_i(Q_i)<C_L^R\end{cases} \tag{23}$$

Here, $APF_U^A-APF_L^A$, $ADC_U^A-ADC_L^A, W_U^A-W_L^A$ and $C_U^A-C_L^A$ are the maximum acceptable violation of the aspiration levels APF_U^A and ADC_L^A, W_L^A, C_L^A respectively. Similarly, $APF_U^R-APF_L^R$, $ADC_U^R-ADC_L^R, W_U^R-W_L^R$ and $C_U^R-C_L^R$ are the maximum acceptable violation of the rejection levels APF_L^R and ADC_U^R, W_L^R, C_L^R respectively. The pictorial representation of Membership and Non-membership functions of Average Profit, Average Cost, Available space and Available budget are given in Figure-1 to Figure-4 respectively.

Now the Intuitionistic fuzzy inventory problem is transformed to the following problem:

$$\begin{aligned}
&Max(\mu_{APF}(Q_i),\mu_{ADC}(Q_i),\mu_W(Q_i),\mu_C(Q_i)),\\
&Min\ (\nu_{APF}(Q_i),\nu_{ADC}(Q_i),\nu_W(Q_i),\nu_C(Q_i))\\
&subject\ to\ \mu_{APF}(Q_i)+\nu_{APF}(Q_i)\le 1\\
&\mu_{ADC}(Q_i)+\nu_{ADC}(Q_i)\le 1,\\
&\mu_W+\nu_W\le 1,\\
&\mu_C+\nu_C\le 1,\\
&\nu_j\ge 0,\ j=APF,ADC,W,C,\\
&\mu_j\ge\nu_j,\ j=APF,ADC,W,C,\\
&Q_i>0, i=1,2,\ldots,n.
\end{aligned} \tag{24}$$

Now, to solve the above problem, covert it to any one of the following form, namely, (a)Intuitionistic Fuzzy Non-linear Programming Problem(IFNLPP) or, (b) Weighted Intuitionistic

Fuzzy Additive Goal Programming Problem (WIFAGPP).

6.1 Intuitionistic Fuzzy Non-linear Programming Problem(IFNLPP)

Here, Min-aggregator is used for conjunction and max-operator is used for disjunction. Then the problem (24) is reduced to the following Intuitionistic Fuzzy Non-linear Programming Problem(IFNLPP):

$$\begin{aligned}
&Max\ \alpha - \beta \\
&subject\ to\ \alpha \le \mu_j,\ j = APF, ADC, W, C, \\
&\beta \ge \nu_j,\ j = APF, ADC, W, C, \\
&\alpha \le \beta, \beta \ge 0, \alpha \ge 0, \alpha + \beta \le 1, \\
&Q_i > 0, i = 1, 2, ..., n.
\end{aligned} \tag{25}$$

where α denotes the minimal acceptable degree of objectives and constraints and β denotes the maximal degree of rejection of objectives and constraints.

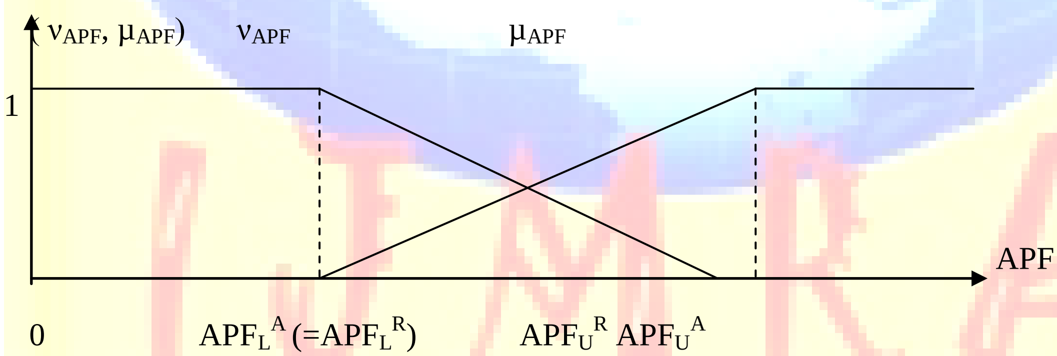
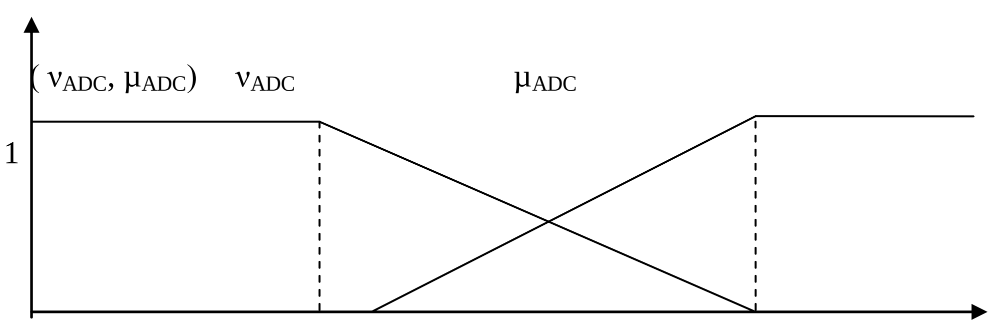


Figure-1: Membership and Non-membership functions of Average Profit



0 ADC_L^A ADC_L^R $ADC_U^R(=ADC_U^A)$ ADC

Figure-2: Membership and Non-membership functions of Average Cost

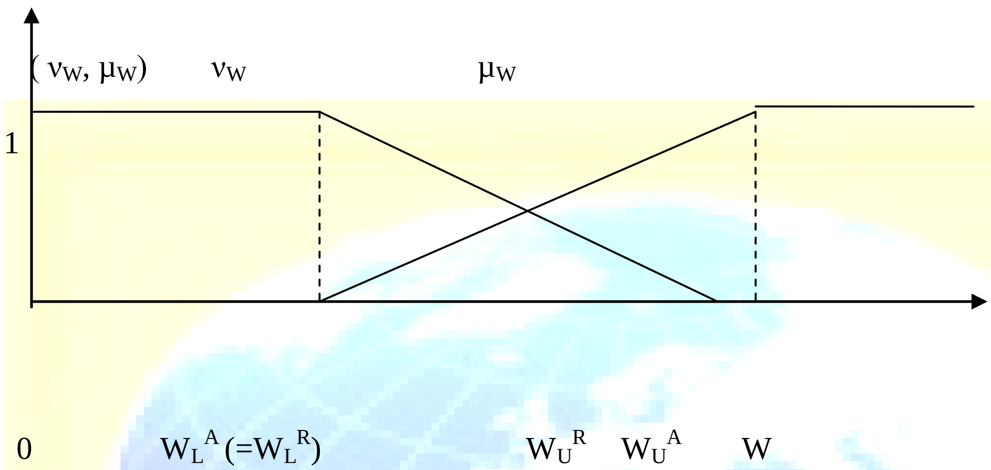


Figure-3: Membership and Non-membership functions of Available space (W)

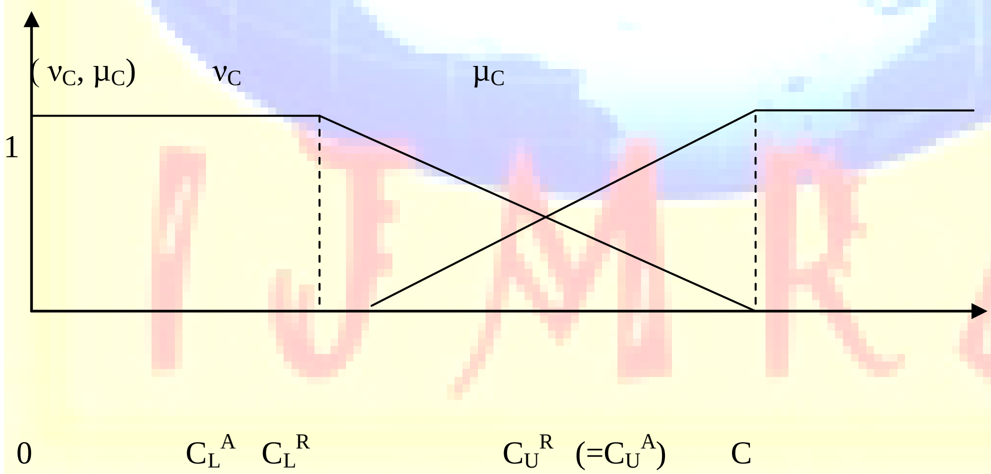


Figure-4: Membership and Non-membership functions of Available budget (C)

6.2 Weighted Intuitionistic Fuzzy Additive Goal Programming Problem (WIFAGPP)

In this case, the above problem (24) reduced to the following crisp optimization problem.

$$
\begin{aligned}
& Max \sum_j w_j(\mu_j - \nu_j), \\
& subject\ to \sum_j w_j = 1, w_j \geq 0, j = APF, ADC, W, C, \\
& \mu_j \geq \nu_j, j = APF, ADC, W, C, \\
& Q_i > 0, i = 1, 2, ..., n.
\end{aligned}
\tag{26}
$$

Here $w_j \in [0,1], j = APF, ADC, W, C$ are called weights. Values of different weights are prescribed by the decision maker according to his / her target. It is to be noted here that the weights corresponding to membership function and non-membership function for a particular objective are assumed to be equal, but it may be different also.

7 Solution Algorithm

To solve intuitionistic fuzzy inventory problem (8), following algorithm is used.

Step-1: Solve the multi-objective non-linear inventory problem (8) as a single objective non-linear programming problem taking only one objective at a time and ignoring the others and obtain the corresponding value of other objective functions. Do this process for all the objective functions. The objective vector, thus obtained consisting of optimum values of each of the objective functions is termed as ideal objective vector.

Step-2: With the help of the result obtained in Step-1, formulate a pay-off matrix as below:

Pay-off Matrix

$$
\begin{array}{cc}
APF(Q_1,Q_2) & ADC(Q_1,Q_2) \\
\end{array}
$$
$$
\begin{bmatrix}
APF^*(Q_1^1,Q_2^1) & ADC(Q_1^1,Q_2^1) \\
APF(Q_1^2,Q_2^2) & ADC^*(Q_1^2,Q_2^2)
\end{bmatrix}
$$

where * indicates the optimum objective value.

Step-3: From the pay-off matrix obtained in Step-2, determine the upper and lower bounds of each of the objectives for the degree of acceptance and degree of rejection as follows:

$$APF_U^A = \max\ (APF(Q_1^1, Q_2^1), APF(Q_1^2, Q_2^2)),$$

$$APF_L^A = \min\ (APF(Q_1^1, Q_2^1), APF(Q_1^2, Q_2^2)),$$

$$ADC_U^A = \max\ (ADC(Q_1^1, Q_2^1), ADC(Q_1^2, Q_2^2)),$$

$$ADC_L^A = \min\ (ADC(Q_1^1, Q_2^1), ADC(Q_1^2, Q_2^2)).$$

It is to be noted that in case of a maximization (minimization) problem, the upper (lower) bound of membership (non-membership) function is always greater than that of non-membership (membership) function. In this case following relations are used.

$$APF_U^R = APF_U^A + k(APF_U^A - APF_L^A) \text{ for } 0 < k < 1$$

and $APF_L^R = APF_U^A + k(APF_U^A - APF_L^A)$ for $k = 0$.

$$ADC_L^R = ADC_L^A + k(ADC_U^A - ADC_L^A) \text{ for } 0 < k < 1$$

and $ADC_U^R = ADC_U^A + k(ADC_U^A - ADC_L^A)$ for $k = 0$.

Also determine the lower and upper bounds of each of the constraint goals.

Step-4: Construct fuzzy programming problem of (24) and find its equivalent problems, namely, Intuitionistic Fuzzy Non-linear Programming Problem (IFNLPP) (25) or Weighted Intuitionistic Fuzzy Additive Goal Programming Problem (WIFAGPP)(26).

Step-5: Solve IFNLPP (25) and / or WIFAGPP (26) by using appropriate mathematical programming algorithm to get an optimal solution and evaluate all objective functions at these optimal compromise solutions.

Step-6: Stop

8 Numerical Example

Let us consider an inventory model in which there is only two items i.e, $n = 2$. The other parameter values are as follows:

$S_1 = \$12, P_1 = \$9, C_{11} = \$1.2, a_1 = 0.03, c_{31} = \$200, \alpha_1 = 100, \beta_1 = 0.3, S_2 = \$10, P_2 = \$6, C_{12} = \$1.4,$
$a_2 = 0.04, c_{32} = \$150, \alpha_2 = 110, \beta_2 = 0.4, A_1 = 0.50\,ft^2, A_2 = 0.75\,ft^2.$

When there is no budgetary constraint as well as space constraint the pay off matrix is as below:

Pay off Matrix

APF	ADC
513.67^*	151.70
0.00	13.68^*

From the pay off matrix, it is seen that maximum value of APF is $\$513.67$ and minimum value of ADC is $\$13.68$. So, we take

$APF_L^A = 100.00$, $APF_U^A = 513.67$, $APF_L^R = 100.00$, $APF_U^R = 500.00$, $ADC_L^A = 13.68$, $ADC_U^A = 200.00$, $ADC_L^R = 14.00$, $ADC_U^R = 200.00$, $W_L^A = 340.00$, $W_U^A = 700.00$, $W_L^R = 350.00$, $W_U^R = 700.00$, $C_L^A = 4000.00$, $C_U^A = 5000.00$, $C_L^R = 4020.00$, $C_U^R = 5000.00$.

8.1 Solution of Intuitionistic Fuzzy Non-linear Programming Problem (IFNLPP)

Using the above numerical data and following the algorithm proposed in Section-7 above, the solution of intuitionistic fuzzy inventory problem (8) is found as $APF = 412.96, ADC = 53.62, Q_1 = 104.59, Q_2 = 396.94$.

8.2 Solution from Weighted Intuitionistic Fuzzy Additive Goal Programming Problem (WIFAGPP)

With the help of same numerical data and following the algorithm proposed in Section-7 above, the solution of intuitionistic fuzzy inventory problem (8) is found as $APF = 392.55, ADC = 52.88, Q_1 = 92.92, Q_2 = 404.72$.

Solution of WIFAGPP for different weights

$(w_{APF}, w_{ADC}, w_W, w_C)$	APF	ADC	Q_1	Q_2
(0.25,0.25,0.25,0.25)	392.55	52.88	92.92	404.72
(0.20,0.20,0.30,0.30)	359.66	51.68	79.25	413.83
(0.20,0.30,0.20,0.30)	357.24	51.93	78.42	414.39

(0.20,0.30,0.30,0.20)	389.89	52.79	91.61	405.59
(0.30,0.20,0.20,0.30)	394.33	92.93	93.81	404.13
(0.30,0.20,0.30,0.20)	419.48	53.90	109.12	393.92
(0.30,0.30,0.20,0.20)	417.58	53.82	107.75	394.83

9 Conclusion

When the degree of rejection (nonmembership) is defined simultaneously with degree of acceptance (membership) of the objectives and when both of these degrees are not complementary to each other, then Intuitionistic Fuzzy sets can be used as a more general tool for describing uncertainty. In this paper, Solution technique for Intuitionistic Fuzzy Non-linear Programming (IFNLPP) is used to solve multi-objective inventory problem.

Also, a new technique to transfer multi-objective intuitionistic fuzzy optimization problem to a crisp one, namely, Weighted Intuitionistic Fuzzy Additive Goal Programming Problem (WIFAGPP) in an intuitionistic fuzzy environment is introduced in this paper and is used it to solve multi-objective inventory problem. Effective solutions are offered for multi-objective inventory decision making problem. Here, using different set of weights decision maker may find different intuitionistic fuzzy pareto optimal solutions and accept one or more of them which is / are more suitable to him / her according to the nature of physical environment. Again, to solve the inventory model linear membership and non-membership functions are used here, although one can use non-linear membership and non-membership functions of hyperbolic, parabolic or any other types.

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